## Exercise 2

For the hydrogen atom if $\lambda > 0$, why would you expect equation (4) not to have a solution that satisfies the condition at infinity?

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**Solution**

Equation (4) in the textbook is

$$-R_{rr} - \frac{2}{r}R_r - \frac{2}{r}R = \lambda R, \tag{4}$$

and the condition at infinity that $R$ has to satisfy is

$$\int_0^\infty |R(r)|^2 r^2 \, dr < \infty.$$

In the limit as $r \to \infty$, the second and third terms on the left side of equation (4) tend to zero.

$$-R_{rr} = \lambda R$$

If $\lambda > 0$, then the general solution in the limit can be written in terms of sine and cosine.

$$\lim_{r \to \infty} R(r) = C_1 \cos(\sqrt{\lambda}r) + C_1 \sin(\sqrt{\lambda}r)$$

Because sine and cosine do not vanish as $r \to \infty$, the condition at infinity cannot be satisfied.